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Does the Phugoid Frequency Depend on Speed?

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Introduction

THE phugoid frequency is thought to be inversely proportional to the forward speed, although this has never been proved. Three counterexamples to this conviction are presented. Subsequently, it is shown that if the speed alone is varied, keeping the aerodynamic and thrust derivatives fixed by some means, the phugoid frequency remains unchanged, establishing that the phugoid frequency is independent of speed. The widespread belief of the dependency of the phugoid frequency on speed perhaps results from the fact that, in most instances, the aerodynamic derivatives vary with speed in such a manner as to make the phugoid time period roughly proportional to the speed.

The notation of Roskam[1] is used. The thrust derivatives are assumed to be combined with the aerodynamic derivatives; i.e., X_u stands for $X_u + X_{Tu}$, M_u stands for $M_u + M_{Tu}$, and M_α stands for $M_\alpha + M_{T\alpha}$.

Analysis

The earliest approximation to the phugoid mode was put forth by Lanchester,[2] who derived the following expression for phugoid frequency:

$$\omega_p = g\sqrt{2}/U_1 \tag{1}$$

This simple result suggests the following:

Conjecture A. The phugoid frequency is inversely proportional to the forward speed.

Data from flight tests are often in harmony with Conjecture A. In the absence of more accurate theory, it is widely believed that Conjecture A is true. This is stated in almost every text on atmospheric flight dynamics, often substantiated with a numerical example or two.

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Table 1 Counterexamples to Conjecture A

Aircraft	Page no. in Ref. 3	Altitude	Mach number	ω_p, rad/s
F-104	51	Sea level	0.257	0.1520
			0.800	0.0504
			1.100	0.0523
		35,000 ft	0.600	0.0709
			0.900	0.0839
Convair 880 A	202	35,000 ft	0.700	0.0528
			0.800	0.0538
			0.860	0.0504

At the outset, counterexamples to this conjecture are shown. Consider the data in Table 1 from Heffley and Jewell.[3]

The phugoid frequency increases with increase in forward speed, disproving Conjecture A, in three instances in Table 1: when the mach number increases from 0.8 to 1.1 at sea level for the F-104, when the mach number increases from 0.6 to 0.9 at 35,000 ft for the F-104, and when the mach number increases from 0.7 to 0.8 at 35,000 ft for the Convair 880M. These results call for a reexamination of the age-old statement in Conjecture A.

Consider the following expression for phugoid frequency, proposed by the authors in a recent study:[4]

$$\omega_p = \sqrt{\frac{g(M_\alpha Z_u - M_u Z_\alpha)}{M_q Z_\alpha - U_1 M_\alpha}} \tag{2}$$

In Ref. 4, numerical simulations involving 15 cases of various types of aircraft under varying flight conditions were carried out to verify the authenticity of this expression. The data were taken from Appendix C of Roskam's text on flight dynamics.[1] This collection of data pertains to six modern aircraft in a total of 16 flight conditions. The aircraft chosen represent diverse missions: a small four-place transportation airplane, a 19-passenger commuter airliner, a small jet trainer, a medium-size high-performance business jet, a supersonic fighter-bomber, and a large wide-body jet transport. The flight conditions range from power approach at sea level to cruise at medium and high altitudes. The database is thus representative of a wide spectrum of airplanes and flight conditions.

It was shown that the approximate expression (2) does not differ from the exact value by more than 4% in the 15 cases considered. The corresponding variation of Eq. (1) from the exact value for the same 15 cases was shown to be between 0.8% and 52.3%.

Having thus reposed faith in Eq. (2), expand the dimensional derivatives in terms of the nondimensional derivatives to obtain

$$\omega_p = \sqrt{\frac{g\left[C_{m_\alpha}\left(C_{L_u} + 2C_{L_1}\right) - \left(C_{m_u} + 2C_{m_1}\right)\left(C_{L_\alpha} + C_{D_1}\right)\right]}{\left(C_{m_q}\bar{c}/2\right)\left(C_{L_\alpha} + C_{D_1}\right) + (2m/\rho S)C_{m_\alpha}}} \tag{3}$$

The speed U_1 does not appear explicitly in the above equation, proving that the phugoid frequency is independent of the forward speed, provided the aerodynamic and thrust derivatives are held fixed by some means. This may be stated as an alternative conjecture.

Conjecture B. The phugoid frequency is independent of the forward speed, provided the aerodynamic and thrust derivatives are kept constant by some artificial means.

Although the approximate equation (2) is a fairly accurate representation of the phugoid frequency in the test cases of the recent study by the authors,[4] doubts still may persist over its validity when the speed is varied over a large band. Such fears are allayed by a provocative test in which both the approximate and the exact phugoid frequencies are calculated for a range of speeds varying from 50 to 1000 ft/s, far exceeding the flight envelopes of the aircraft in question. The approximate frequency is obtained by solving Eq. (2) and the exact value is obtained by solving the longitudinal characteristic equation

$$As^4 + Bs^3 + Cs^2 + Ds + E = 0 \tag{4}$$

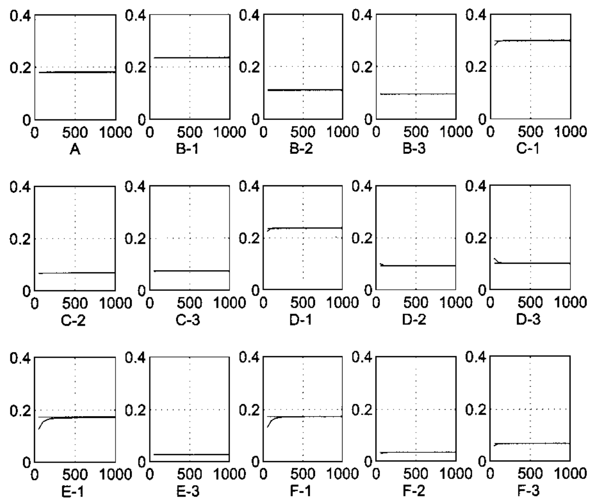

Fig. 1 Phugoid frequency ω_p in rad/s (y axis) vs U_1 in ft/s (x axis).

where

$$A = U_1 - Z_{\dot{\alpha}}$$

$$B = -(U_1 - Z_{\dot{\alpha}})(X_u + M_q) - Z_\alpha - M_{\dot{\alpha}}(U_1 + Z_q)$$

$$\begin{aligned} C = {} & X_u[M_q(U_1 - Z_{\dot{\alpha}}) + Z_\alpha + M_{\dot{\alpha}}(U_1 + Z_q) \\ & + M_q Z_\alpha - Z_u X_\alpha + M_{\dot{\alpha}} g \sin\Theta_1 - M_\alpha(U_1 + Z_q)] \end{aligned} \tag{5}$$

$$\begin{aligned} D = {} & g\sin\Theta_1(M_\alpha - M_{\dot{\alpha}} X_u) + g\cos\Theta_1[Z_u M_{\dot{\alpha}} + M_u(U_1 - Z_{\dot{\alpha}})] \\ & - M_u X_\alpha(U_1 + Z_q) + Z_u X_\alpha M_q + X_u[M_\alpha(U_1 + Z_q) - M_q Z_\alpha] \end{aligned}$$

$$E = g\cos\Theta_1(M_\alpha Z_u - Z_\alpha M_u) + g\sin\Theta_1(M_u X_\alpha - X_u M_\alpha)$$

The results are plotted in Fig. 1 for the 15 different cases. In each plot, the ordinate shows ω_p and the abscissa shows U_1. Each graph is labeled according to the aircraft and the flight condition it represents. For instance, E-3 denotes aircraft E in flight condition 3. Uniformity is maintained in the graphs by ensuring that all of their ordinates vary from 0 to 0.4. The abscissa of each graph varies from 0 to 1000 ft/s.

It can be seen from the figure that the approximation and the exact value are indistinguishable except at the left extremity. Except for the left extremity, Fig. 1 shows that the phugoid frequency is independent of the forward speed. On close scrutiny, it is seen that the region in which there appears to be a discrepancy between the exact and the approximate values lies below 100 ft/s. Introspection reveals that any speed falling below this limit lies below the stalling speeds of all the aircraft considered in the simulation. Thus, as far as practical flight is concerned, the difference between the exact solution to the phugoid frequency and the approximate solution represented by Eq. (2) is immaterial. This strongly supports Conjecture B.

This suggests that it is not speed, but the change of aerodynamic derivatives with speed, that causes the change in the phugoid frequency. In most cases, the aerodynamic derivatives vary in such a manner as to reduce the phugoid frequency when the speed is increased, leading to widespread belief in Conjecture A.

Conclusion

The notion of yore that the phugoid frequency is inversely proportional to the forward speed is disproved. Counterexamples where the phugoid frequency increases as the forward speed is increased are cited. Using an expression for the phugoid frequency developed by the authors, we have shown that the phugoid frequency is independent of the forward speed, provided the aerodynamic and thrust derivatives are held fixed by some artificial means.

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Direct Adaptive Control for Gravity-Turn Descent

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I. Introduction

GRAVITY-TURN descent has been investigated widely[1–4] and used in practice[5,6] for terminal descent to lunar and planetary surfaces. Initial studies of this descent method utilized simple linear feedback control laws to track predefined velocity-range descent profiles. It has been shown that these linear methods can be improved by using nonlinear transformation methods to artificially linearize the system error dynamics.[7] However, all of these control laws require knowledge of key system parameters such as the local gravitational acceleration. In addition, the control laws are not robust to parameter variations or to unmodeled disturbances such as thruster failure. A direct adaptive control law is presented that provides robust tracking of a predefined descent profile. It is shown that the control law is a function only of the lander velocity and altitude and does not require any model-dependent parameters. Because of this inherent robustness, the control law is able to adapt to realistic thruster failure modes such as thruster valves jammed open or closed. Last, because explicit on-line parameter estimation is not required, the control law is computationally efficient, and so it appears attractive for onboard implementation.

II. Gravity-Turn Descent

Gravity turn is a simple descent method whereby the lander thrust vector is aligned opposite to its velocity vector at all points along the descent trajectory. This requirement can be implemented easily onboard by using the attitude control system to null body rates about the lander velocity vector. It can be shown that the method is also near optimal in providing minimum-fuel descents. Although the thrust-vector orientation is fixed, the thrust magnitude can be modulated to track predefined velocity–altitude or velocity–slant range descent profiles. The method also has the useful property that a vertical landing is guaranteed. For terminal descent maneuvers, the planar translational dynamics of the lander may be modeled as a point mass moving over a flat surface with a uniform local gravitational acceleration g, viz.,

$$\dot{v} = ng + g\cos\psi \tag{1a}$$

$$v\dot{\psi} = -g\sin\psi \tag{1b}$$

$$\dot{h} = -v\cos\psi \tag{1c}$$

where the lander state variables are illustrated in Fig. 1. The altitude h can be obtained by a simple radar altimeter and the lander velocity components obtained from a multibeam Doppler radar. These components also provide the lander flight-path angle ψ. The only control

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